Students Use Graphic Organizers to Improve Mathematical Problem-Solving Communications

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“Let me give you a math story problem.” This sentence often strikes fear in many middle grades students as well as some teachers. As international comparisons, national commissions, and state assessment results confirm, students have difficulty solving mathematical applications problems (Lester, 2007; U. S. Department of Education Institute of Educational Science, 2007; TIMMS, 2003; McREL, 2002; National Research Council, 2002; Illinois State Board of Education, 1997).

Improving students’ problem-solving abilities is a major, if not *the* major, goal of middle grades mathematics (National Council of Teachers of Mathematics, 2000; 1995; 1989). To address this goal, the author, who is a university mathematics educator, and nine inner-city middle school teachers developed a math/science action research project. This article describes our unique approach to mathematical problem solving derived from research on reading and writing pedagogy, specifically, research indicating that students who use graphic organizers to organize their ideas improve their comprehension and communication skills (Goeden, 2002 ; National Reading Panel, 2000).

Many teachers and students use graphic organizers to enhance the writing process in all subject areas, including mathematics. Graphic organizers help students organize and then clarify their thoughts, infer solutions to problems, and communicate their thinking strategies.

We designed a classroom action research project to study a problem-solving instructional approach in which students used graphic organizers. Our goal was to improve student achievement in three areas of our state’s math assessment in open-response problems: mathematics knowledge, strategic knowledge, and mathematical explanation. In this article, we discuss graphic organizers and their potential benefits for both students and teachers, we describe the specific graphic organizer adaptations we created for mathematical problem solving, and we discuss some of our research results of using the *four corners and a diamond* graphic organizer.

Benefits of using graphic organizers in mathematics learning

A graphic organizer is an instructional tool students can use to organize and structure information and concepts and to promote thinking about relationships between concepts. Furthermore, the spatial arrangement of a

This article reflects the following *This We Believe* characteristics: Active Learning — Multiple Learning Approaches — Varied Assessments

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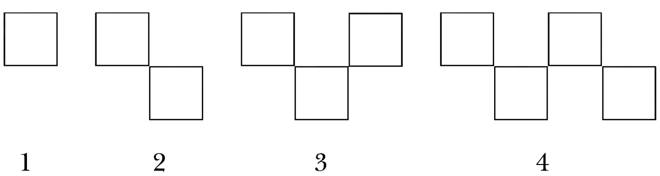
graphic organizer allows the student, and the teacher, to identify missing information or absent connections in one's strategic thinking (Ellis, 2004).

Middle grades teachers already use many different types of graphic organizers in the writing process. All share the common trait of depicting the process of thinking into a pictorial or graphic format. This helps students reduce and organize information, concepts, and relationships. When a student completes a graphic organizer, he or she does not have to process as much specific, semantic information to understand the information or problem (Ellis, 2004). Graphic organizers allow, and often require, the student to sort information and classify it as essential or non-essential; structure information and concepts; identify relationships between concepts; and organize communication about an issue or problem.

Consider the following middle grades math problem from a recent state assessment.

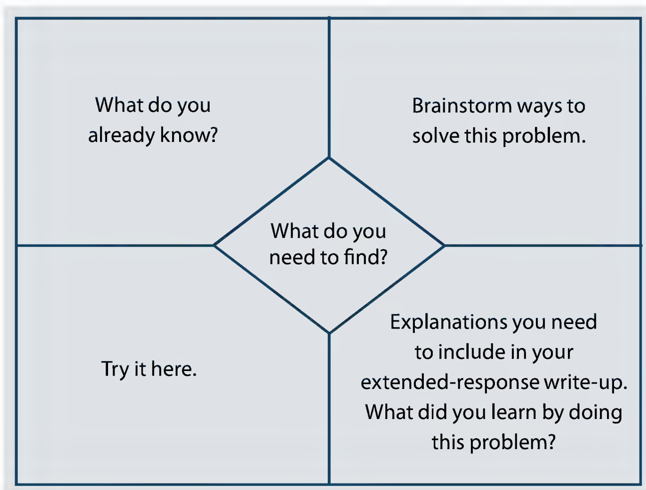
> How many vertices (corners) are there in 1, 2, 3, 4, 5, 6 … n squares when they are arranged in the following way?



What did you first think when reading the problem? Did you first think of the meaning of the term "vertices" or that this is a mathematical pattern problem? Did you first think of counting the corners or that this looks like an arrangement of tables? Did you first think to discuss in your solution why you are not just adding four with every square? Did you first try to think of the singular form of the word vertices?

Initial thinking is not a linear activity, especially in mathematical problem solving. Yet, the result of problem solving—the written solution—often looks like a linear, step-by-step procedure. Good problem solvers brainstorm different thoughts and ideas when first presented with a problem, and these may or may not be useful. Problem solvers can use a graphic organizer to record random information but not process it. A student can later reflect upon usefulness of the information and ideas. If the information and ideas help the student make relationships between concepts, then they are essential. A graphic organizer allows a student to quickly organize, analyze, and synthesize one's knowledge, concepts, relationships, strategy, and communication. It also gives every student a starting point for the problem-solving process.

Figure 1 Four corners and a diamond mathematics graphic organizer

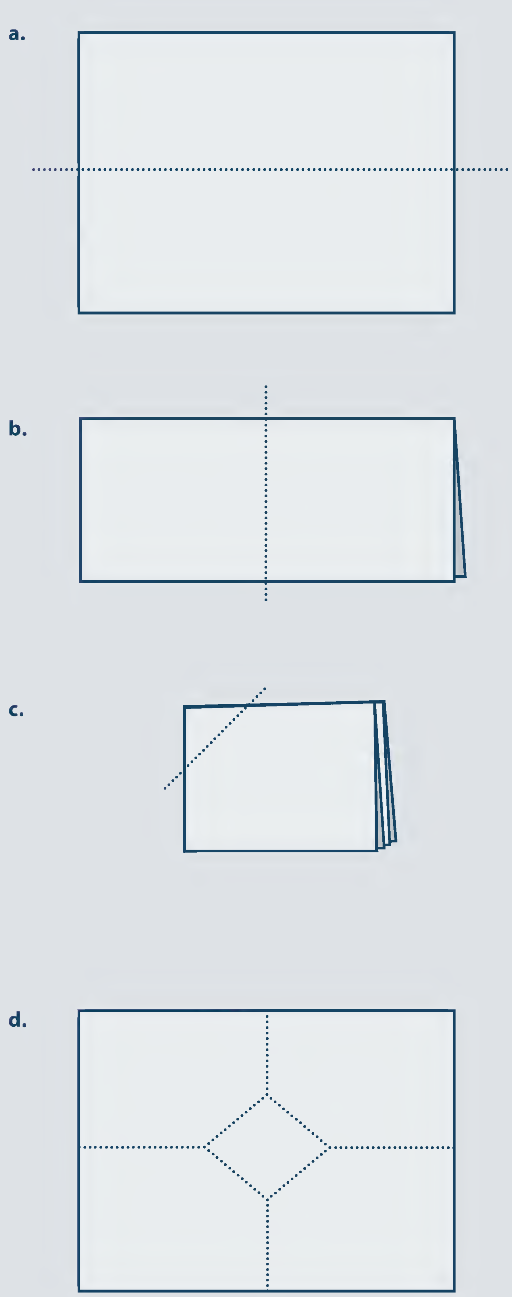


Adapting a graphic organizer for mathematical problem solving

Figure 1 depicts the four corners and a diamond graphic organizer. This graphic organizer was modified from the four squares writing graphic organizer described by Gould and Gould (1999). The four square writing method is a formulaic writing approach, originally designed to teach essay writing to children in a five paragraph, step-by-step approach. The graphic organizer portion of the method specifically assists students with prewriting and organizing. We saw beneficial problem-solving aspects in the graphic organizer portion of this writing method for mathematics.

Figure 2 Four corners and a diamond folding template

Instructions: First, (a) fold paper horizontally, then (b) fold vertically, finally (c) fold the inner corner up. When unfolded you get the four corners areas and the "diamond" in the center as shown in (d).



Our four corners and a diamond graphic organizer has five areas:

1. What do you need to find?
2. What do you already know?
3. Brainstorm possible ways to solve this problem.
4. Try your ways here.
5. What things do you need to include in your response? What mathematics did you learn by working this problem?

Actually, the form in Figure 1 does not have to be given to the students each time. Figure 2 shows how students, using a blank piece of paper, make the four corners and a diamond graphic organizer template. The student folds the paper into fourths, first folding the paper horizontally ("hot dog style"), then vertically ("hamburger style"), and finally the inner corner is folded up. When the paper is unfolded, the creases form the four corners and the "diamond" rhombus in the middle. The teachers reported that students later (e.g., during state testing) often folded or drew the five areas on their paper to begin problem solving.

So how does the use of the four corners and a diamond graphic organizer differ from the traditional Polya's four-step mathematical problem-solving hierarchy? In terms of objectives, it does not. Obviously, the four corners and a diamond graphic organizer is designed to help students understand the problem, devise a plan, carry out the plan, and look back (Polya, 1944). However, by having the non-linear layout of the graphic organizer, the student is not expected to do these "steps" in a hierarchical, procedural order that some students misapply. It is the implementation process, how students form their response, that is the important aspect of the four corners and a diamond graphic organizer (Zollman, 2006a).

The pictorial orientation allows students to record their ideas in whatever order they occur. If students first think of the unit for their final answer, then this is recorded in the fifth, bottom-right area. This idea (the unit), then, is not needed in the short-term memory because a reminder is recorded. If students first think of a possible procedure for their answer, this is recorded in the third, upper-right area. The four corners and a diamond graphic organizer allows, and even encourages, students to use their problem-solving strategies in a non-hierarchical order. A student can work in one area of the organizer and later work a different area. It also shows that completing a problem-solving response has several different, but related, aspects.

Students do not begin writing a response until some information or ideas are in all five areas. The four corners and a diamond graphic organizer especially encourages students to begin working on a problem before they have an identified solution method. As in the four square writing method, the students then organize and edit their thoughts by writing their solution in the traditional linear response, using connecting phrases and adding details and relationships. The steps for the open response write-up are as follows: (1) state the problem; (2) list the given information; (3) explain methods for solving the problem; (4) identify mathematical work procedures; and (5) specify the final answer and conclusions.

The graphic portion of the organizer allows all students to fill in parts of the solution process. It encourages all students to persevere—to "muck around" working on a problem. Further, teachers quickly can identify where students are confused when solving a problem by simply examining the graphic organizer.

The teacher should model proper use of the four corners and a diamond graphic organizer and have students work in groups when introducing this tool. Working in groups allows students to see that many problems can be worked in more than one way and that different people start in different places when solving a problem. In their small-group discussions, students identify relationships between the areas in the graphic organizer and among the various solutions.

Graphic organizers can benefit students when they take standardized state mathematics assessments, specifically open-response problem-solving items. Most states use a scoring rubric for these types of items. In Illinois, for example, the scoring rubric has three categories: mathematical knowledge, strategic knowledge, and explanation (Illinois State Board of Education, 2005). Responses are scored on a four-point scale for each category, with scores ranging from zero for "no attempt" to four for "complete." Typically, low-ability students do not attempt to show any work in one or more response categories, while average-ability students often have disorganized responses. Higher-ability students sometimes skip steps in their explanations. The four corners and a diamond graphic organizer helps each type of student produce a more complete response in each of the three categories and, thus, receive a higher score.

Four corners and a diamond provides students with a logical framework for writing about problem-solving tasks.

Impact of graphic organizers

Nine middle school teachers decided to use the open-response mathematics questions as the focus of their action research on the effects of using graphic organizers. Teachers administered pre- and post-tests with their students to see if using the four corners and a diamond graphic organizer impacted their performance.

All teachers reported dramatic improvements in students' mathematics scores on open-response items after implementing the four corners and a diamond graphic organizer. The percentage of students (N=186) who scored at the "meets" or "exceeds" levels on each of the open-response item categories on the pre-test was 4% for math knowledge, 19% for strategic knowledge, and 8% for explanation. After instructing students to use

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the graphic organizer in mathematical problem solving, the percentage of students scoring "meets" or "exceeds" on the post-test improved to 75% for math knowledge, 68% for strategic knowledge, and 68% for explanation (Zollman, 2006a; 2006b).

Each teacher self-collected and self-scored these data using the state's scoring rubric. Overall scores increased from a 27% average on the pre-test to a 70% average on the post-test. Data collected, analyzed, and triangulated from three sources—the teachers, the action research pre- and post-test data, and the students' work—suggests that the use of the graphic organizer in mathematical problem solving may significantly help students coordinate their mathematical ideas, methods, thinking, and writing. The graphic organizer helped students coordinate various parts of mathematical problem solving: (a) What is the question? (b) What information is known? (c) What strategies might be used? (d) Which operations, procedures, or algorithms of the strategy need to be shown? (e) What explanations and reflections are needed to communicate the method(s) of solution? (Zollman, 2006a; 2006b).

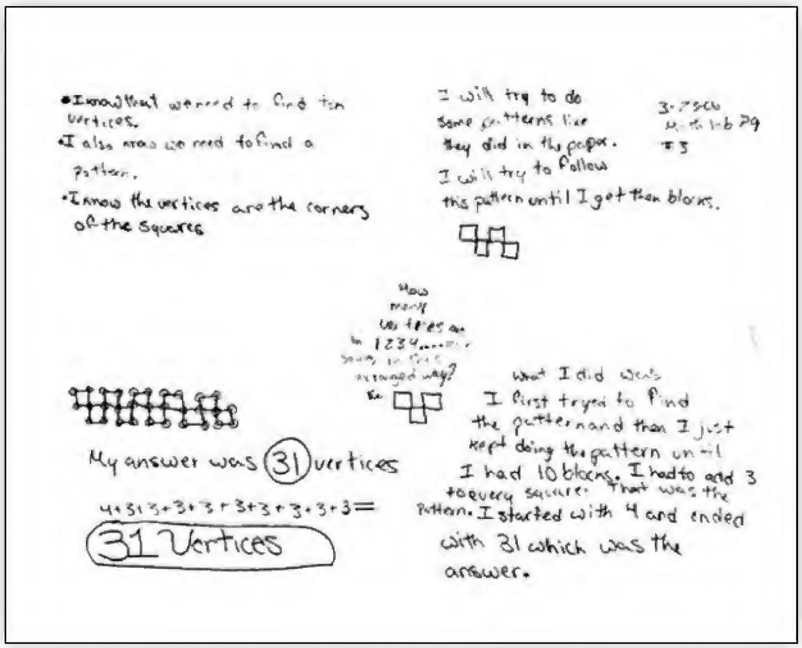
Graphic organizers help students communicate their thinking when they solve problems.

The teachers found the use of graphic organizers in mathematical problem solving to be very efficient and effective for all levels of students. The teachers saw that their lower-ability students, who normally would not have attempted problems, had now written partial solutions. The organizer appeared to help average-ability students organize thinking strategies and help high-ability students improve their problem-solving communication skills (Zollman, 2006b). Students now had an efficient and familiar method for writing and communicating their thinking in a logical argument.

Samples of students' work

The samples of student work in Figures 3 and 4 are from an open-response squares and vertices problem before and after the use of graphic organizers in the classroom.

Sample 1 shows the work of a student who was presented the problem before becoming familiar with the four corners and a diamond graphic organizer. Sample 2 shows the same student's work later in the semester, after learning how to problem solve using the graphic organizer. The student's strategy on the pre-test was to count the individual vertices in the picture, then add these numbers. This work shows a misunderstanding of the problem, limited strategy, and no explanation. On the post-test, this same student's work shows a complete understanding of the problem presented (10 squares) and a complete explanation of a correct strategy that will transfer to other problems, however, it lacks a concluding algebraic formula to demonstrate

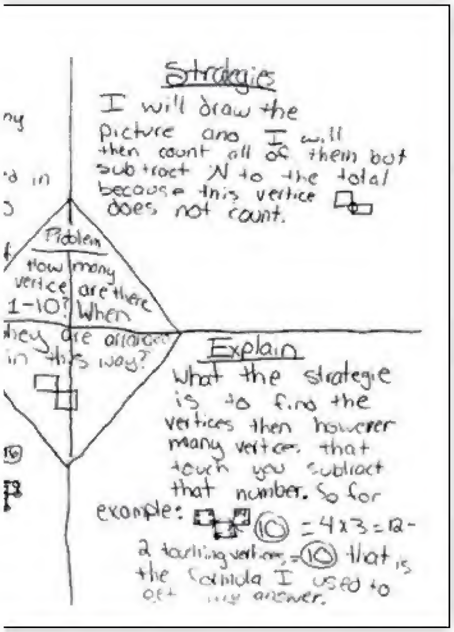


Sample 2

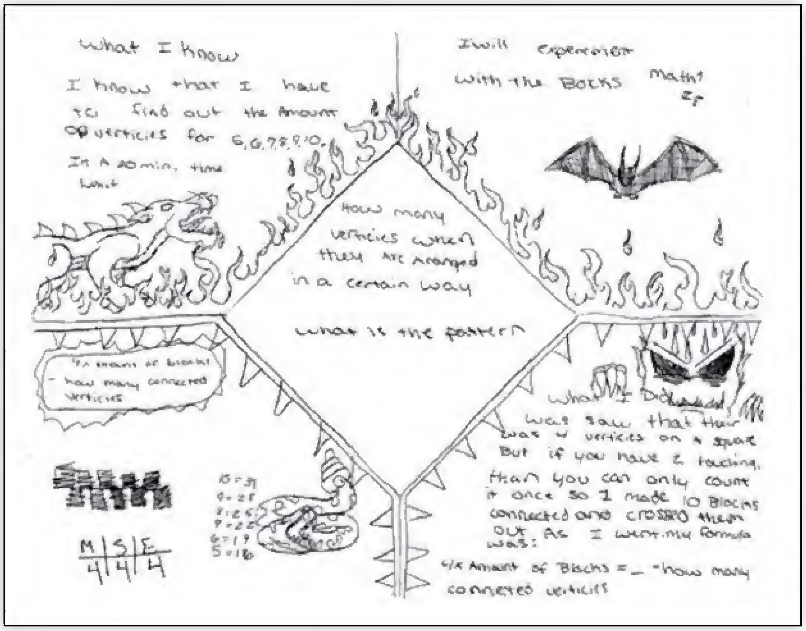
mathematical knowledge. While it is not a perfect response, understanding, organization, development, and reflection are all strongly represented on the graphic organizer.

The second student's pre-test (Sample 3) shows the common incorrect strategy of just counting the total vertices in the picture. It appears that the student then attempted to "add" the individual pictures in the student's own drawing to again count the vertices. However, without any explanation, the teacher cannot know what strategy, if any, the student was attempting. Again, this work shows a misunderstanding of the problem, limited strategy, and no explanation. This student's post-test (Sample 4) illustrates excellent understanding, organization, development and reflection of the problem presented (10 squares). The graphic organizer shows the student's complete, correct strategy, solution, and explanation of the problem. For mathematical knowledge, the formula is well explained in words, not as an algebraic expression. This would be acceptable on state assessments, as the problem did not specifically ask for an algebraic expression.

Sample 5 is the post-test work of a higher-ability student. This student's work demonstrates a full understanding of the problem, a correct solution, and a complete explanation. The drawings also suggest that



Sample 4



Sample 5

the student feels a sense of ownership of and satisfaction with the solution and probably finished the problem with plenty of time to spare.

Caveats

We hoped the students in our action research study would improve their problem solving with an instructional intervention from pre-test to post-test; however, no single instructional method directly affects learning. Rather, instruction is one of many factors that may influence learning. Others include the curriculum, the student, the class, and the teacher. Nevertheless, the teachers who conducted the action research described in this article believed the graphic organizer was associated with many of the positive outcomes in their students' problem-solving ability (Zollman, 2006b).

The crucial factor in the effectiveness of any instructional method is how it is implemented. If four corners and a diamond graphic organizer is used as a linear, systematic procedure to teach problem solving, it will succeed sporadically. In fact, any direct teaching about problem solving is likely to have intermittent success. Giving students a chart of Polya's (1944) four steps in problem solving or a graphic organizer sheet may help students learn the steps of problem solving. However, students may remain uncertain about where to start a problem, confused by essential versus non-essential information, or unaware how to communicate important steps and reflections in their solutions. We found that graphic organizers aid students in all three of these areas.

Allowing students to first use their own thinking–and then reflect, revise, and re-organize their knowledge, strategies, and communication–helps them improve their problem-solving abilities. Initially, teaching about problem solving as a hierarchy of procedural steps is neither efficient nor effective. Our results confirm other studies that found teaching *via* problem solving is the key instructional process (Lester, 2007).

Summary

As our work suggests, effective reading and writing strategies like graphic organizers may have crossover effects in mathematics for students of all ability levels. We found that four corners and a diamond, when properly used, was an extremely useful instructional method in the middle grades mathematics classroom. Our instructional approach helped students construct content knowledge and strategic knowledge and, we contend, it also improved their mathematical communication skills. In addition, four corners and a diamond allowed teachers to quickly identify the weaknesses and strengths of students' problem solving abilities. As teachers seek to expand and improve students' mathematical knowledge to help them solve problems, they may find that good teaching in reading and writing is good teaching in math.

Extensions

The author shows how graphic organizers that are typically used to help students organize their thoughts while writing in ELA can also be used to help them think through problem-solving tasks in mathematics.

How can teachers use graphic organizers to actively engage students in thinking and problem-solving activities in all areas of the curriculum?

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References

Ellis, E. (2004). *What's the big deal about graphic organizers?* Retrieved July 7, 2003, from http://www.Graphic Organizers.com

Goeden, J. (2002). *Using comprehension frames (graphic organizers) to impact students' reading comprehension*. Unpublished thesis. Black Hills State University.

Gould, J., & Gould, E. (1999). *Four square writing method for grades 1–3*. Carthage, IL: Teaching and Learning Company.

Illinois State Board of Education. (2005). *Mathematics scoring rubric*. Springfield, IL: Author.

Illinois State Board of Education. (1997). *Illinois learning standards*. Springfield, IL: Author.

Lester, F. (Ed.). (2007). *Second handbook of research on mathematics teaching and learning*. Greenwich, CT: Information Age.

Mid-continent Research for Education and Learning (McREL). (2002). *ED thoughts: What we know about mathematics teaching and learning*. Aurora, CO: Author.

U.S. Department of Education Institute of Educational Sciences. (2007). *The nation's report card*. Retrieved January 28, 2008, from http://nces.ed.gov/nationsreportcard

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Reading Panel. (2000). *Teaching children to read: An evidence-based assessment of the scientific research literature on reading and its implications for reading instruction.* Washington DC: U.S. Department of Health and Human Services.

National Research Council. (2002). *Learning and understanding: Improving advanced study of mathematics and science in U.S. high schools.* Washington DC: National Academy Press.

Polya, G. (1944). *How to solve it.* Garden City, NY: Doubleday.

Trends in International Mathematics and Science Study (TIMSS). (2003). *Trends in international mathematics and science study.* Washington, DC: International Association for the Evaluation of Educational Achievement.

Zollman, A. (2006a, April). Four-corners is better than Four-squares: Assessment connection for solving mathematics story problems. Presented at the Annual Conference of the National Council of Teachers of Mathematics, St. Louis, MO.

Zollman, A. (2006b, October). Write is right: Improving students' problem solving using graphic organizers. Presented at the 105th Annual Convention of the School Science and Mathematics Association, Missoula, MT.

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